

Difference-in-Differences

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Introduction: DiD and panel data

- *Difference-in-differences* (*DiD*) is one of the most popular strategies for **estimating the average treatment effect** of some policy or shock.
- To use DiD, we need at least two time periods: one before the treatment and one after.
 - This requires *panel data*.
- In *panel data* (also called longitudinal data), we observe a number of individuals across many years.
 - Eg: National Longitudinal Survey of Youth, Prices of different stocks across diff years, etc.

Motivation

- We want to estimate the effect of a policy/shock across groups.
- However, the policy assignment is not necessarily uncorrelated with group characteristics. (Ie, *random assignment* fails)
- How can we **identify the effect** of the policy **without being confounded by the individual-level differences**?

Canonical DiD

In the canonical DiD model, we have:

- **Two periods**: treatment occurs (for some units) in period 2
- Identification of the *average treatment effect of the treated* (*ATT*) from **parallel trends** and **no anticipation**

Setup

- **Panel data** on Y_{it} for $i = 1, \dots, N$ and $t = 1, 2$
- **Treatment timing**: Some units ($D_i = 1$) are treated in period 2; everyone else is not ($D_i = 0$)

Setup

- **Panel data** on Y_{it} for $i = 1, \dots, N$ and $t = 1, 2$
- **Treatment timing**: Some units ($D_i = 1$) are treated in period 2; everyone else is not ($D_i = 0$)
- **Potential outcomes**: $Y_{it}(1)$ for the treated individuals and $Y_{it}(0)$ for the non-treated. In period 2, we can only observe:

$$Y_{i2} = Y_{i2}(1)D_i + Y_{i2}(0)(1 - D_i).$$

- Estimand: ATT (Average Treatment effect of the Treated)

$$\tau_{ATT} = \mathbb{E}[Y_{i2}(1) - Y_{i2}(0) | D_i = 1]$$

Key identifying assumptions

Parallel trends:

$$E[Y_{i2}(0) - Y_{i1}(0) | D_i = 1] = E[Y_{i2}(0) - Y_{i1}(0) | D_i = 0]$$

- In the absence of the treatment, the Y_{it} across units evolve in parallel. That is, individuals/units may have different levels, but their changes would evolve in parallel.

Key identifying assumptions

Parallel trends:

$$E[Y_{i2}(0) - Y_{i1}(0) | D_i = 1] = E[Y_{i2}(0) - Y_{i1}(0) | D_i = 0]$$

No anticipation: $Y_{i1}(1) = Y_{i1}(0)$

- Intuitively, outcome in period 1 is NOT affected by treatment status in period 2
- This assumption is often left implicit in notation, but important for interpreting DiD estimand as a causal effect in period 2.

Identification

Under **parallel trends** and **no anticipation**,

$$\tau_{ATT} = (\mathbb{E}[Y_{i2} | D_i = 1] - \mathbb{E}[Y_{i1} | D_i = 1]) - (\mathbb{E}[Y_{i2} | D_i = 0] - \mathbb{E}[Y_{i1} | D_i = 0])$$

- The first difference is Change for treated.
- The second difference is Change for untreated/control.
- [Figure illustration online](#)

In plain words, the estimand of interest (ATT) is equal to the “difference-in-differences” of population means.

Estimation

1. Use sample analogs:

$$\hat{\tau}_{DiD} = (\bar{Y}_{12} - \bar{Y}_{11}) - (\bar{Y}_{02} - \bar{Y}_{01}) = \Delta Y_{1t} - \Delta Y_{0t}$$

- Intuitively, we generate a counterfactual for the treatment using the changes in the untreated units: $E(Y_{i1} - Y_{i0} | D_i = 0)$.

Estimation

2. Equivalently, $\hat{\tau}_{DiD}$ is equal to the OLS coefficient $\hat{\beta}$ from

$$Y_{it} = \alpha_i + \phi_t + \beta D_{it} + \epsilon_{it}$$

where $D_{it} = 1$ if $D_i = 1$ and $t = 2$; otherwise, $D_{it} = 0$.

- $\hat{\tau}_{DiD}$ is sometimes referred to as the *Two-way Fixed Effects estimator (TWFE)*. That is, the setup includes both unit fixed effects (α_i) and time fixed effects (ϕ_t).

Cases of DiD

1 treatment timing, Binary treatment, 2 periods

- Card and Krueger (AER, 1994)

1 treatment timing, Binary treatment, T periods

- Yagan (AER, 2015)

1 treatment timing, Continuous treatment

- Berger, Turner and Zwick (JF, 2020)

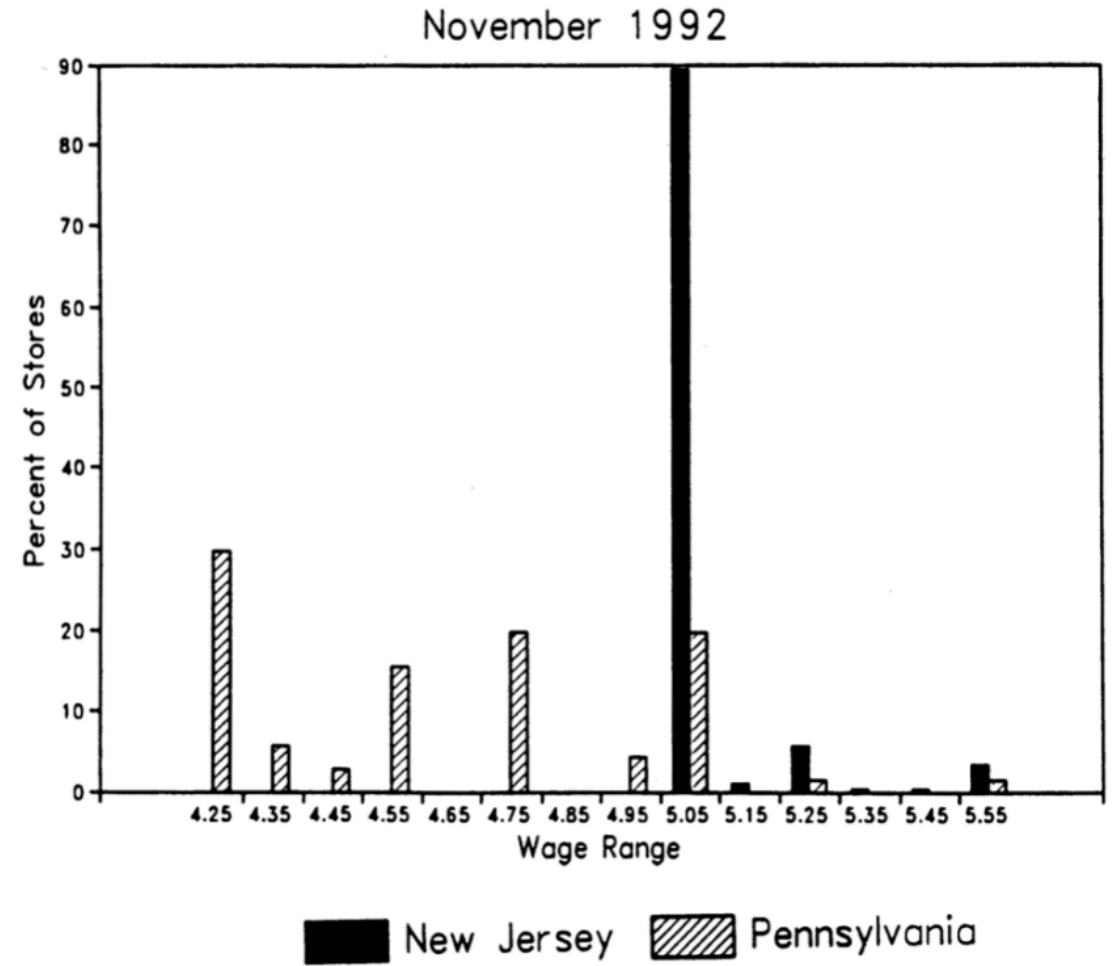
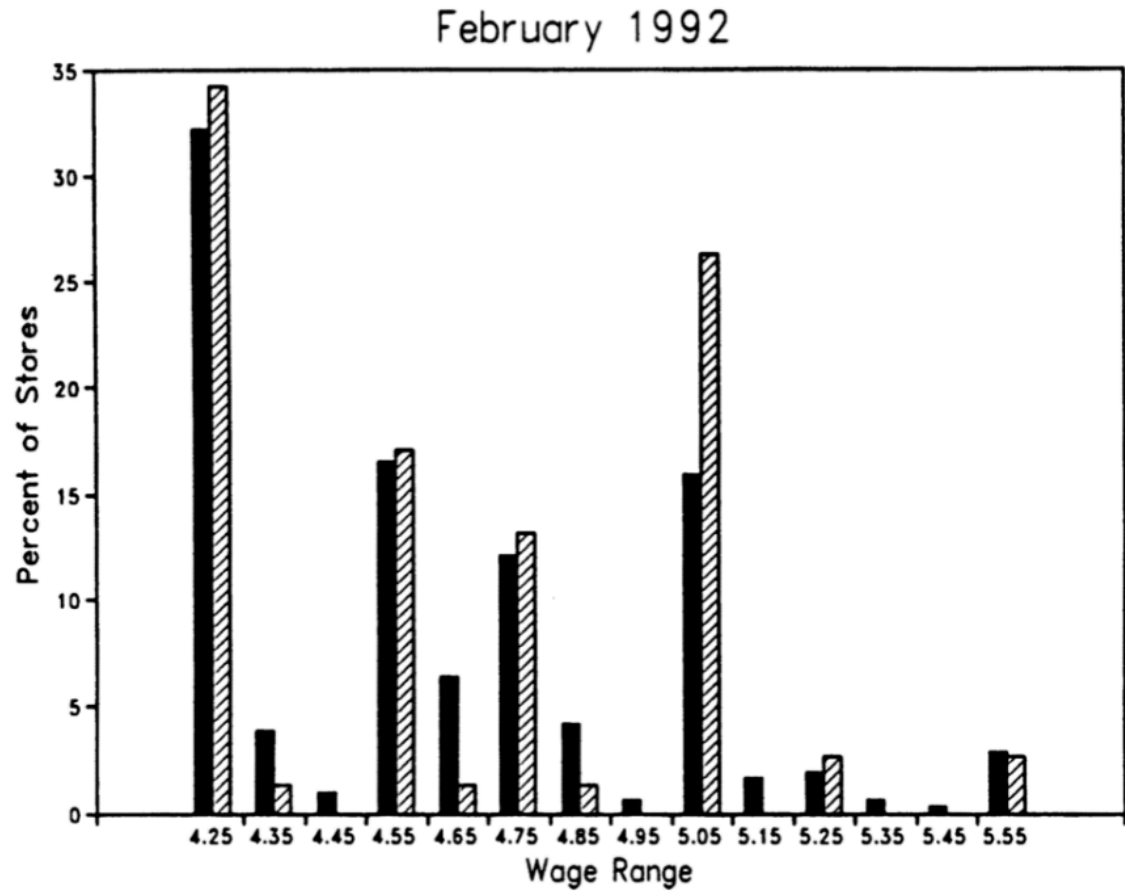
Staggered treatment timing, Binary treatment

- Bailey and Goodman-Bacon (AER, 2015)

Card and Krueger (1994)

- Card and Krueger (1994) study the impact of New Jersey increasing the minimum wage 4.25 to 5.05 dollars an hour on April 1, 1992
- Key question is what impact does this have on employment?
 - Need a counterfactual for NJ, and use Pennsylvania as a control
- Collected data in 410 fast food restaurants
 - Called places and asked for employment and starting wage data
 - Sample data from Feb 1992 and Nov 1992
- So D_i is NJ vs PA, and $t = 1$ is Feb 1992 and $t = 2$ is Nov 1992.

Stark Effect on Wages in Card and Krueger (1994)



Effect on Employment in Card and Krueger (1994)

Despite a large increase in wages, **no negative impact on employment**

- In fact, marginally significant positive impact

Looking at raw data, this positive impact is driven by a decline in PA

- This decline is reasonable if you think that PA is a good counterfactual, since 1992 is in the middle of a recession

A second comparison can be run with stores whose starting wage in pre-period was above treatment cutoff

- These stores perform similarly to PA

Key considerations for thinking about Card and Krueger (1994)

- The treatment can't really be thought of as randomly assigned
 - Treatment is completely correlated within states
 - As a result, any within-state correlation of errors will be correlated with treatment status
- Given the limited number of states, time periods, and treatments, more valuable to view this as a case study.